Coherent electron beams and sources

1989 J. Phys.: Condens. Matter 1 3737

(http://iopscience.iop.org/0953-8984/1/23/022)

J. Phys.: Condens. Matter **1** (1989) 3737–3742. Printed in the UK

LETTER TO THE EDITOR

Coherent electron beams and sources

N Garcia† and H Rohrer‡

† Departamento de Fisica de la Materia Condensada, C-III, Universidad Autonoma de Madrid, Cantoblanco, 28049-Madrid, Spain

‡ IBM Research Division, Zurich Research Laboratory, Säumerstrasse 4, CH-8803 Rüschlikon, Switzerland

Received 19 January 1989, in final form 14 April 1989

Abstract. We show that the point-source electron beams recently obtained by Fink have a spatial coherent length of the order of centimetres. The narrow opening of the beam suggests that the source itself is also coherent.

The study and understanding of new materials and processes in condensed-matter physics requires high-resolution and versatile microscopic techniques and, hence, instruments. Of these, electron microscopes are among the most extensively used instruments in different kinds of laboratories. A handicap is the difficulty involved in obtaining coherent electron beams (CEB). One impact of high coherence is the realisation of the initial ideas of Gabor [1] regarding holography and interferometry with atomic resolution. Note especially the developments in the optics of these two techniques with the discovery of the laser. In electron microscopy, there have been advances [2] in this direction by using field-emission tips [3] of 500–1000 Å radius as emitters of the electrons. The main two advantages of the field-emission sources (FES) are the partial or total prevention of space-charge-induced electron–electron interaction known as the Boersch effect [4] and partial coherence with reasonable brightness. These FES have resulted in extensive development of electron interferometry (see Missiroli and co-workers [5] for a review) as well as electron holography, where the experiments of Tonomura and co-workers and Lichte [6] have been extended to a large variety of condensed-matter problems.

Recently, Fink [7] has produced tips terminating in a well characterised geometry of a few atoms to be used in scanning tunnelling microscopy (STM) [8]. He showed [9] that these tips are a realisation of point sources (PS) of electron and ion beams with remarkable properties, two of which initiated this investigation, namely: (i) a current of a few μA can be obtained by applying a small voltage around 200 V between the tip of the electrode and the screen separated by $L = 15$ cm, and (ii) the beam opening is only a few degrees, as deduced from a spot size Δ_s of the electron beam on the screen of a few mm.

The purpose of this Letter is to show that these PS produce a highly coherent electron beam with a spatial coherent length l_t of the order of centimetres and that these PS are also coherent sources.

In a first step we show that from electrostatic potential considerations alone, for the experimental conditions, the beam opening is an order of magnitude larger than in the

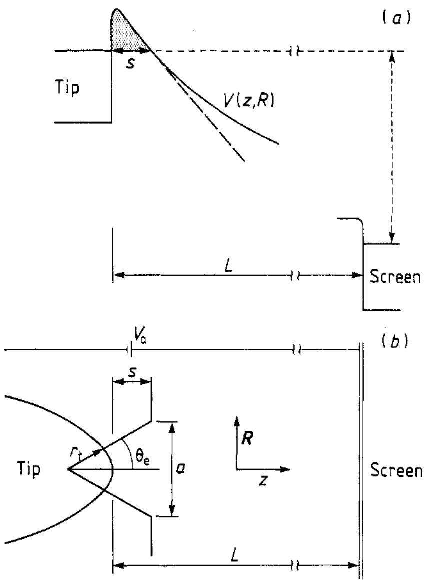

Figure 1. Schematic potential energy configuration (a) and geometry (b) of tip and screen electrodes. a is the diameter of the beam, where the potential $V(z, R)$ cuts the Fermi energy at a distance s and defines the emission disc.

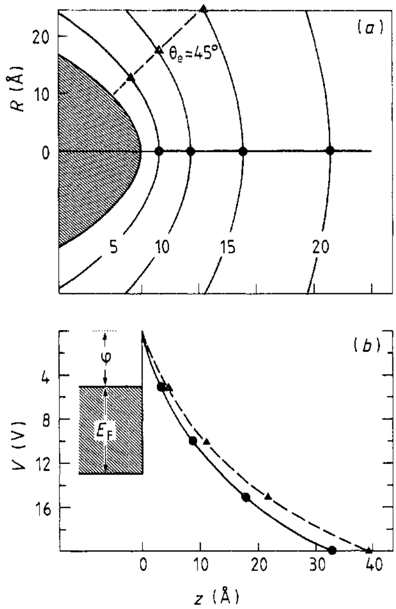

Figure 2. (a) Equipotential lines (indicated on the curves in volts), and (b) potential drop for a hyperboloid tip with an apex curvature r_t = 10 Å and with the centre at the screen at a distance L = 15 cm and a potential of 200 V. At the escape angle, θ_e (~45°), the current density has decreased by a factor of e by using Simmons' formula [10].

experiment. This result does not depend critically on the current intensity, nor on the voltage applied between tip and screen, nor on the tip radius. To obtain tunnel currents of μA, for typical work functions of a few eV and a tip radius of ~10 Å, near-tip fields of ~1 V Å^{-1} are required, which in turn provides effective tunnel distances s of a few angstroms and an area of the disc that emits electrons of approximately 100 Å^2 from the tip. These values follow straightforwardly from Simmons' formula [10] for field-emission currents and are also in agreement with numerical calculations performed for STM experiments [11]. Figure 1 depicts the emission and tunnel regions.

Calculations [12] of the voltage drop near the tip for different geometries (semi-sphere, conical, ellipsoid and hyperboloid) show that to produce fields ~1 V Å^{-1} with V_a = 200 V and L = 15 cm, the tip requires a radius (r_t = ~10 Å) of atomic dimensions in good agreement with experiments [8]. We point out that the voltage drop is not only a function of r_t but of the specific geometry of the tip and naturally of L. Figure 2 shows the equipotential lines and voltage drop for a hyperboloid tip which can provide the tunnel barriers adequate for microamp currents. We have introduced the image force correction [13] and found that microamp currents can be obtained from a tip radius of ~20 Å with fields of ~0.5 V Å^{-1} for the case of the hyperboloid. The escape angle θ_e, i.e., the angle formed by the tip axis and the direction where the current density decreased by a factor of e (see figure 2) is about 45°. The general result of the calculations with various geometries was that the value of the escape angle can only be substantially reduced at the expense of a great increase in the tunnel distance s and therefore a drastic

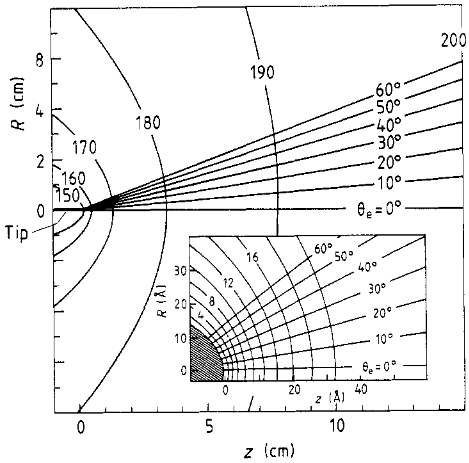

Figure 3. Trajectories, T, for the same geometry as in figure 2 for electrons at various escape angles and initial velocity. The tip is indicated but is not drawn to scale. Inset: an enlargement of the tip region. Note that the trajectories do *not* follow the field lines. In both main figure and inset, the equipotentials are given in volts.

reduction of the tunnel current. Considering the classical electron trajectories, the calculated beam opening is 40° (see figure 3). Even reducing the current intensity by three orders of magnitude gives only a slightly smaller beam opening of about 35°, still an order of magnitude larger than that observed. We therefore conclude that the electrostatic potential cannot focus the beam for the experimental current densities.

We now proceed to the spatial coherence of the beam and the source, which also offers an explanation for the beam opening observed. We treat this problem in a manner analogous to the propagation of a scalar radiation wave [14] from a plane source of size a. Consider a quasi-monochromatic emitting source of wavelength λ in the plane $\boldsymbol{R}$ of the emission disc with mutual coherence function $\Gamma(\boldsymbol{R}_1, \boldsymbol{R}_2)$. Its propagation in the far zone (Fraunhofer region), $L \gg 2a^2/\lambda$, which in our case is well satisfied ($L = 15$ cm, $a = 10$ Å, and $\lambda = 1$ Å), is given by

$$\Gamma(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2) = \iint \mathrm{d}\boldsymbol{R}_1\, \mathrm{d}\boldsymbol{R}_2 \exp[\mathrm{i}k(\boldsymbol{\varphi}_1 \cdot \boldsymbol{R}_1 - \boldsymbol{\varphi}_2 \cdot \boldsymbol{R}_2)]\Gamma(\boldsymbol{R}_1, \boldsymbol{R}_2) \tag{1}$$

where $k = 2\pi/\lambda$ and $\boldsymbol{\varphi}_1$ and $\boldsymbol{\varphi}_2$ are the usual angular coordinates [14].

If we make the assumption that the source is quasi-homogeneous, then

$$\Gamma(\boldsymbol{R}_1, \boldsymbol{R}_2) = (I(\boldsymbol{R}_1))^{1/2}(I(\boldsymbol{R}_2))^{1/2}\, g(\boldsymbol{\rho}) \tag{2}$$

where $\boldsymbol{\rho} = \boldsymbol{R}_1 - \boldsymbol{R}_2$ and $I(\boldsymbol{R})$ is the intensity of the source at the point $\boldsymbol{R}$ and g is the correlation function at the source. The intensity of the source in the far zone is $J(\varphi) = \Gamma(\varphi, \varphi)$.

The width of the observed spot, Δ_s, and the spatial correlation length l_t at the screen can be obtained from the integral (1) once the values of $I(\boldsymbol{R})$ and $g(\boldsymbol{\rho})$ are known.

We assume that the functions $I(\boldsymbol{R})$ and $g(\boldsymbol{\rho})$ are given by Gaussian distributions of widths a and b, respectively, where b is the spatial coherence length at the source

$$I(\boldsymbol{R}) = \exp(-R^2/2a^2) \qquad g(\boldsymbol{\rho}) = \exp(-\rho^2/2b^2). \tag{3}$$

This may reasonably describe the electrons emitted by field emission. By changing variables

$$\boldsymbol{\alpha} = \boldsymbol{\varphi}_1 - \boldsymbol{\varphi}_2 \qquad \boldsymbol{\beta} = \tfrac{1}{2}(\boldsymbol{\varphi}_1 + \boldsymbol{\varphi}_2) \tag{4}$$

integral (1) becomes

$$\Gamma(\boldsymbol{\alpha}, \boldsymbol{\beta}) = \exp[-\alpha^2(ka)^2/4]\exp\{-\beta^2/[2(kb)^{-2} + (ka)^{-2}]\} \tag{5}$$

and the intensity $J(\boldsymbol{\beta})$

$$J(\boldsymbol{\beta}) = \exp\{-\beta^2/[2(kb)^{-2} + (ka)^{-2}]\}. \tag{6}$$

In order to study the spatial coherence length l_t at the screen (far zone), it is convenient to discuss the value of $\Gamma(\boldsymbol{\alpha}, \boldsymbol{\beta})$ normalised by the intensity

$$\mu(\boldsymbol{\alpha}) = \Gamma(\boldsymbol{\alpha}, \boldsymbol{\beta})/(J(\boldsymbol{\beta} + \boldsymbol{\alpha}/2))^{1/2}(J(\boldsymbol{\beta} - \boldsymbol{\alpha}/2))^{1/2}. \tag{7}$$

From formulae (5) and (6), we have

$$\mu(\boldsymbol{\alpha}) = \exp[\![-[\alpha^2(ka)^2/4]\{1 - 1/[1 + 2(a/b)^2]\}]\!]. \tag{8}$$

Let us discuss the widths at the screen of the spot intensity, Δ_s, and of the coherence function, l_t, as a function of source parameters a and b. These are given for the HFW α_0 and β_0 of $J(\beta)$ and $\mu(\alpha)$ with $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ being the angular coordinates between the line connecting a spot in the screen and the source. They can be related to the width $\alpha_0 = l_t/2L$ and $\beta_0 = \Delta_s/2L$ and therefore

$$\begin{aligned} l_t &= 2L[\![[(ka)^2/4]\{1 - 1/[1 + 2(a/b)^2]\}]\!]^{-1/2} \\ \Delta_s &= 2L[2(kb)^{-2} + (ka)^{-2}]^{1/2}. \end{aligned} \tag{9}$$

Assume incoherent radiation [14]†, $b \sim \lambda$, (which may be the case of a FES source). Then $b/a \ll 1$, and from equation (9) we obtain

$$\begin{aligned} \Delta_s^i &\sim (\sqrt{2}/\pi)(\lambda L/b) \simeq (\sqrt{2}/\pi)L \\ l_t^i &\sim (2/\pi)(\lambda L/a) \end{aligned} \tag{10}$$

i.e., the intensity width Δ_s^i spread approximately over a diameter L. On the other hand, even if the radiation is fully incoherent at the source, its propagation will increase the coherent length [14]. Note that for usual FES, and assuming the same work function in the entire emission area, $\lambda < 0.1$ Å and $a \sim 500$ Å, therefore $l_t^i \sim 2 \times 10^{-4}L$ while in the case of PS, $\lambda = 1$ Å, $a = 10$ Å, $l_t^i \sim 0.1L$. Therefore, even in the case of an incoherent source, the coherence is increased by a factor of 500, because for PS the source is much smaller and λ much larger than for FES.

Table 1 summarises the relevant parameters for FES and PS sources as well as the correlation lengths and spot width at the screen calculated using equations (10) and (11). The main conclusion that may be reached is that the correlation length of the beam increases as the source size is reduced and does not depend much on the correlation length at the source. This is at variance with the behaviour of the spot width, which is much reduced by increasing the coherence of the source. Also, a completely incoherent source has a delta function correlation and a correlation length of zero. However, this is physically impossible since the minimum correlation length is the wavelength of the radiation. Thus we have to consider $b \simeq l$ for incoherent sources. In analogy, the maximum correlation length that can be obtained is in the case where the source is coherent and therefore the correlation length of a coherent source has been taken to be equal to the source size a.

† In principle an incoherent source has a coherent length that tends to zero, however it is reasonable to think that the minimum coherence length is the wavelength of the radiation.

Table 1. Relevant parameters defining the FES and PS sources as well as the correlation lengths and spot intensity widths obtained at the screens for both sources. The values of l_t^c and Δ_s^c for FES sources are not given since these sources are incoherent.

	FES	PS
Electron wavelength, λ (Å)	~ 0.1	~ 1
Source diameter, a (Å)	~ 500	~ 10
Source correlation length, b (Å)		
Coherent sources $b \sim a$	~ 500	~ 10
Incoherent sources $b \sim \lambda$	~ 0.1	~ 1
Correlation length at the screen		
Incoherent sources, l_t^i	$2 \times 10^{-4} L$	$\sim 0.1 L$
Coherent sources, l_t^c	—	$\sim 0.1 L$
Spot width at the screen		
Incoherent sources, Δ_s^i	$\sim L$	$\sim L$
Spot width at the screen		
Coherent sources, Δ_s^c	—	$\sim L/20$

The experimental beam width at screen $\Delta_{s,\exp} \simeq 0.5$ cm $\ll L = 15$ cm is in contradiction to the above prediction, $\Delta_s^i \simeq L$, for an incoherent source. However, the calculated beam width will be drastically reduced if we simply assume that the source has a coherence length b of the order of the source, a, i.e., $b \simeq a$. We then obtain

$$\begin{aligned} \Delta_s^c &\sim (\sqrt{3}/\pi)(\lambda L/a) \\ l_t^c &\sim (\sqrt{6}/\pi)(\lambda L/a). \end{aligned} \tag{11}$$

This may imply that the electrons are emitted *one at a time from a very specific energetic level* and with its wave-packet transversal width equal to the size of the source.

At present we do not have a precise knowledge of the energy levels of such a small emitting object. In our opinion, however, the levels with coherent emission may be defined by the geometry of the constriction which is defined by the tip, near the Fermi level and with large momentum in the direction of emission to have a dominant tunnelling probability. Also the tunnel barrier defined by the applied field may play an important role in defining the levels that contribute to the current. Work is in progress to understand this key point better.

In the PS case $\lambda \sim 1$ Å, $a = 10$ Å, $L = 15$ cm, thus $\Delta_s^c = 0.8$ cm $\ll \Delta_s^i$ and $l_t^c \simeq 0.1L \simeq l_t^i$ (see table 1). As the value of l_t is basically determined by the dimension of the source, in PS a is small and therefore the coherence is large. The value Δ_s^c is in good agreement with the experiment [9]. We take this as an indication that the source has a coherent length $b \sim a$. Note that the expression for Δ_s^c is the same as the diffraction of a coherent wave by a circular aperture of diameter a.

The analysis we have performed is valid assuming that the mean free path of the emitted electrons is larger than the dimensions of the source. When this is not the case one can expect inelastic processes to occur in the sources that break its coherence. This may happen at high emitted currents that can significantly increase the source temperature.

It should be pointed out that the above optical treatment does not apply exactly to our problem. Firstly, the wavelength decreases as the electron moves from the tip to the screen, and secondly, the classical electron trajectory is curved owing to curved field lines. Numerical calculations [12] show that the effective wavelength is within a few percent of that at the screen. Physically this can be understood because ~75% and ~90% of the potential fall within 1 mm and 10 mm, respectively, from the tip. Likewise they show that the electron trajectories are almost straight over most of the path (see figure 3) with a constant wavelength.

The brightness of the beam (electrons per area and solid angle) can be made a factor of 10^4 to 10^6 larger for a point source than for a field-emission source. This is because all the emitted electrons can be used for the beam compared to only some 10^{-4} in the case of a FES, and the beam can have a virtual image of an area about 10^2 smaller. This increase in brightness will increase the number of interference fringes by a factor of some 10^3.

Finally we want to point out that the same kind of analysis also holds for ions. In the expressions of equation (11), the smaller wavelength is partially compensated by the smaller emission area. For keV the ions $\lambda = 0.01$ Å, so with $a = 2$ Å, we obtain $l_t^c = 0.7$ mm and $\Delta_s^c = 0.5$ mm, in reasonable agreement with experiment.

In conclusion we have shown that point-source beams are highly coherent electron and ion beams with a spatial coherence length of the order of centimetres and that the narrow opening of the beam seems to suggest the coherence length of the source is of the size of the source. These coherent electron beams (CEB) can represent a breakthrough in electron holography and interferometry that could visualise very distinctively atomic objects in three dimensions. A direct measurement of the coherence length could be provided by an interferometric Young experiment.

We should like to acknowledge fruitful discussions and computations with L Escapa and J J Sáenz, and discussions on optical coherence with M Nieto-Vesperinas. We are indebted to H-W Fink for commenting on his experiment prior to publication.

References

[1] Gabor D 1949 *Proc. R. Soc.* A **454** 197; 1951 *Proc. Phys. Soc.* B **64** 449
[2] Crewe A V, Eggenberger D N, Wall D N and Welter L N 1968 *Rev. Sci. Instrum.* **576** 39
Crewe A V, Wall J and Langmore J 1970 *Science* **168** 1338
[3] Müller E W 1937 *Z. Phys.* **106** 541
[4] Boersch H 1954 *Z. Phys.* **139** 115
[5] Missiroli F G, Pozzi G and Valdrè U 1981 *J. Phys. E: Sci. Instrum.* **14** 649
[6] Tonomura A 1987 *Rev. Mod. Phys.* **59** 639
Lichte H 1986 *Ultramicrosc.* **20** 293
Zeitler E 1979 *Proc. Meeting 37th Electron Microscopy Society of America* ed. G W Bailey (Baton Rouge, LA: Claitor) p 376
[7] Fink H-W 1986 *IBM J. Res. Dev.* **30** 460
[8] Binnig G and Rohrer H 1986 *IBM J. Res. Dev.* **30** 355
[9] Fink H-W 1988 *Phys. Scr.* **38** 260; private communications
[10] Simmons J 1963 *J. Appl. Phys.* **34** 1793
[11] Garcia N 1986 *IBM J. Res. Dev.* **30** 533
[12] Serena P, Escapa L, Sáenz J J, Garcia N and Rohrer H 1988 *Proc. STM '88 Conf. (Oxford) 1988; J. Microsc.* at press
[13] Gomer R 1961 *Field Emission and Field Ionization* (Cambridge, MA: Harvard University) p 10
[14] Goodman J W 1985 *Coherence of Optical Waves in Statistical Optics* (New York: Wiley) ch 5, p 157
Spense J C H 1981 *Experimental High Resolution Microscopy* (Oxford: Clarendon)